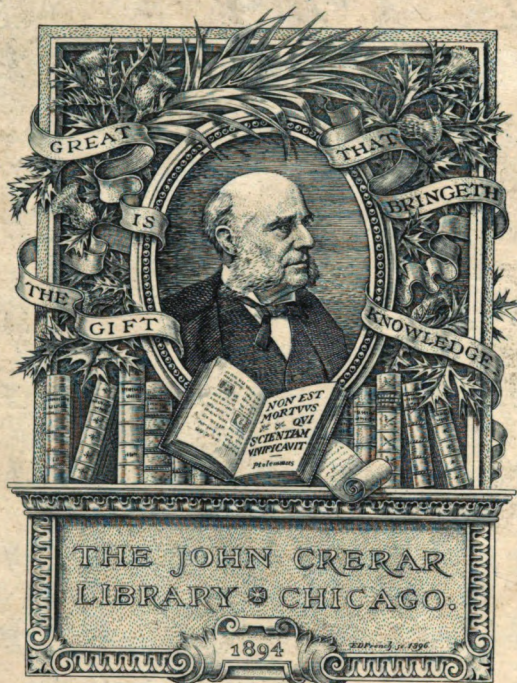


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THE STUB-TOOTH GEAR

THE FELLOWS GEAR SHAPER CO.
SPRINGFIELD, VERMONT

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THE STUB-TOOTH GEAR

EXPLAINING THE ADVANTAGES
OF THE COMBINATION OF THE
SHORT GEAR TOOTH WITH AN
INCREASED ANGLE OF OBLIQUITY

THE FELLOWS GEAR SHAPER CO
SPRINGFIELD VERMONT U S A

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WITH the constantly increasing use of gears for transmitting power in widely varying quantities, the question of the correct shape and size of gear teeth becomes of far greater importance than ever before. It is not sufficient that a gear be well cut and the teeth properly spaced but the shape and proportions of the tooth itself must be carefully considered.

The two most important features are the securing of the nearest approach to a rolling action that it is possible to obtain and the strongest tooth that will meet this condition. The first includes easy running, reduces the friction to the lowest point and consequently has the least wear in action.

We are apt to think of the present standard gear tooth as one of the fixed laws of mechanics and that it represents the highest development possible in this line just as we consider the question of threads fixed by the Sellers or United States Standard form of thread.

But this is not the case and while there is always a perfectly justifiable reluctance about departing from such a recognized standard as the gear tooth it is at least good policy to know both sides of the question and see if the advantages claimed for the newer form of tooth can be backed up with positive proof.

The so-called standard tooth of today is the involute curve with the $14\frac{1}{2}$ degree angle of obliquity or pressure angle but its use is not as universal as might be supposed and the newer form known as the "Stub-tooth" with its 20 degree pressure angle is growing into favor faster than many realize.

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But before discussing this in any way it will be best to trace briefly the development of the regular form of tooth and the causes that led up to its adoption; then to see what reason, if any, exists for a radical change.

The first that were worthy of the name were of epicycloidal form and measured by circular pitch and, as the state of the mechanic arts had not advanced to the point of cutting teeth in a machine, the gears were in most cases made with cast teeth in the foundry. The rest were cast without teeth and these were made of wood and driven into the rim by the old time mill-wright.

With the epicycloidal system it was necessary, in order to avoid the excessive under cutting of the flank of the tooth, to adopt certain proportions of form and length of tooth, which gave continuous action when the gear of 12 teeth was used. Proportions for the length of tooth and other dimensions, that were originated at this time, when the science of applied mechanics was in its infancy, have been handed down to the present time with little change, regardless of the fact that with the dropping of the epicycloidal form of tooth, the conditions have been entirely changed.

The use of interchangeable gearing in sets of milled gears, was made possible by the invention of special machinery by the late Mr. F. A. Pratt, but this was only developed along the line of the epicycloidal form of tooth as the advantages of the involute system were not appreciated at that time.

The adoption of the involute form of tooth, together with the diametral pitch system of

measurement, has been gradual but constant and it has now practically superseded all other forms.

It will be evident to any one making a study of the application of gearing to the various classes of work called for by modern engineering that the commonly accepted standard of form and proportions of gear teeth are not the best for the purposes required.

With the epicycloidal system of gear teeth the mean of the angle of obliquity was about 15 degrees, and in determining the proportions of the interchangeable involute gearing, this angle of rack was adopted as the basis of the system. One point that had much weight in deciding upon the angle was the fact that the sine of the angle of $14\frac{1}{2}$ degrees was .25, a proportion which was easy for the wheel-wright to lay out and so the angle of $14\frac{1}{2}$ was decided upon. So strongly has this angle become impressed on the mechanic that it has become a revered tradition of gearology that a greater angle than this cannot be used with any degree of success because of excessive friction and the consequent wear on bearings.

While this idea may have been more or less correct with the proportions of shafts and bearings in use at that time, with its cast gears and epicycloidal teeth, it surely has very little weight under present conditions and especially in these days of generously proportioned bearings. The effect of the angle of obliquity on the wear of bearings has been unduly exaggerated and as a consequence, the proportions of gearing in general use today are open to great improvement in the three essential points of strength, durability and running qualities.

With an interchangeable set of gears the length of the tooth should be sufficient to give an arc of action equal to the pitch arc, even with the smallest pinions. This will keep one pair of teeth in contact until the next pair is in position to take up the load. There is therefore a fixed relation between the length of tooth and the angle of obliquity. In ordinary practice the number of teeth of the pinion is limited to 12, and the length of the tooth ordinarily adopted is such that the action is continuous.

It is, however, a fallacy to argue that the teeth should be as long as possible, with the idea that a gain is made if two or more pairs of teeth are in mesh at once. Conditions are never such that an equal division of the load is possible, and a length of tooth beyond that which is necessary to insure a continuity of action, produces undue friction and wear. The best results are obtained when the arc through which the action of a pair of teeth takes place is from $1\frac{1}{4}$ to $1\frac{1}{2}$ times the pitch arc.

Although the subject of gearing has been investigated and discussed at great length by many mechanics, there is one phase of the subject that has received but scant attention, and this is the co-relative effect of the angle of obliquity, or pressure angle, and the length of the tooth upon the efficiency and wearing qualities of the tooth itself. It can be shown that an excessive sliding action takes place at a certain portion of the tooth action between gears having teeth of the standard or $14\frac{1}{2}$ degree angle, and that by increasing this angle it is possible to so shorten the tooth that only such portions of the curve are used as will give nearly a complete

rolling action. An increase in the angle of obliquity has often been advocated by others and is advantageous, but its advantages are very limited if a tooth of the standard length is retained.

The use of gears for the transmission of power in automobiles has perhaps called attention to this question more than any other line of work and to show clearly just what this action really is, the following diagrams have been prepared for comparison of the action of the Stub-tooth with that of the $14\frac{1}{2}$ degree angle, using as an

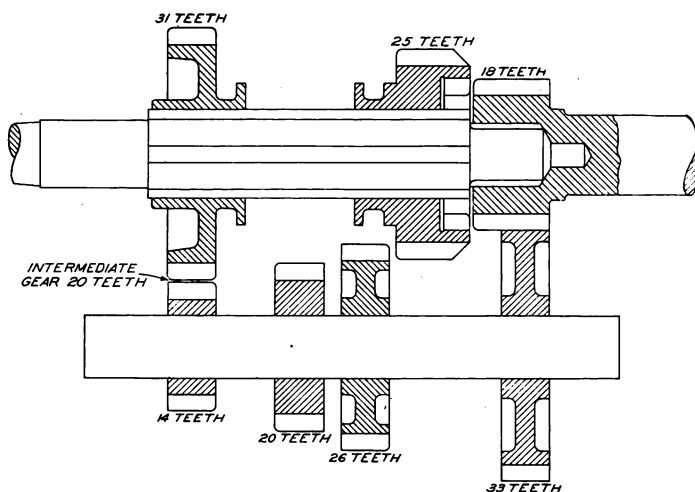


Fig. 1
A typical
Automobile
Transmission
Gear.

example the gears of the sliding transmission of an automobile.

The following gears have been selected as being typical of such a transmission or "gear box."

Low Gear	18 - 33
2nd "	26 - 25
3rd "	20 - 31
Reverse	14 - 20 - 31

These gears are shown assembled in Figure 1. As the combinations 18 - 33 and 20 - 31 are very similar, the second has been omitted from the comparisons and the others will now be considered.

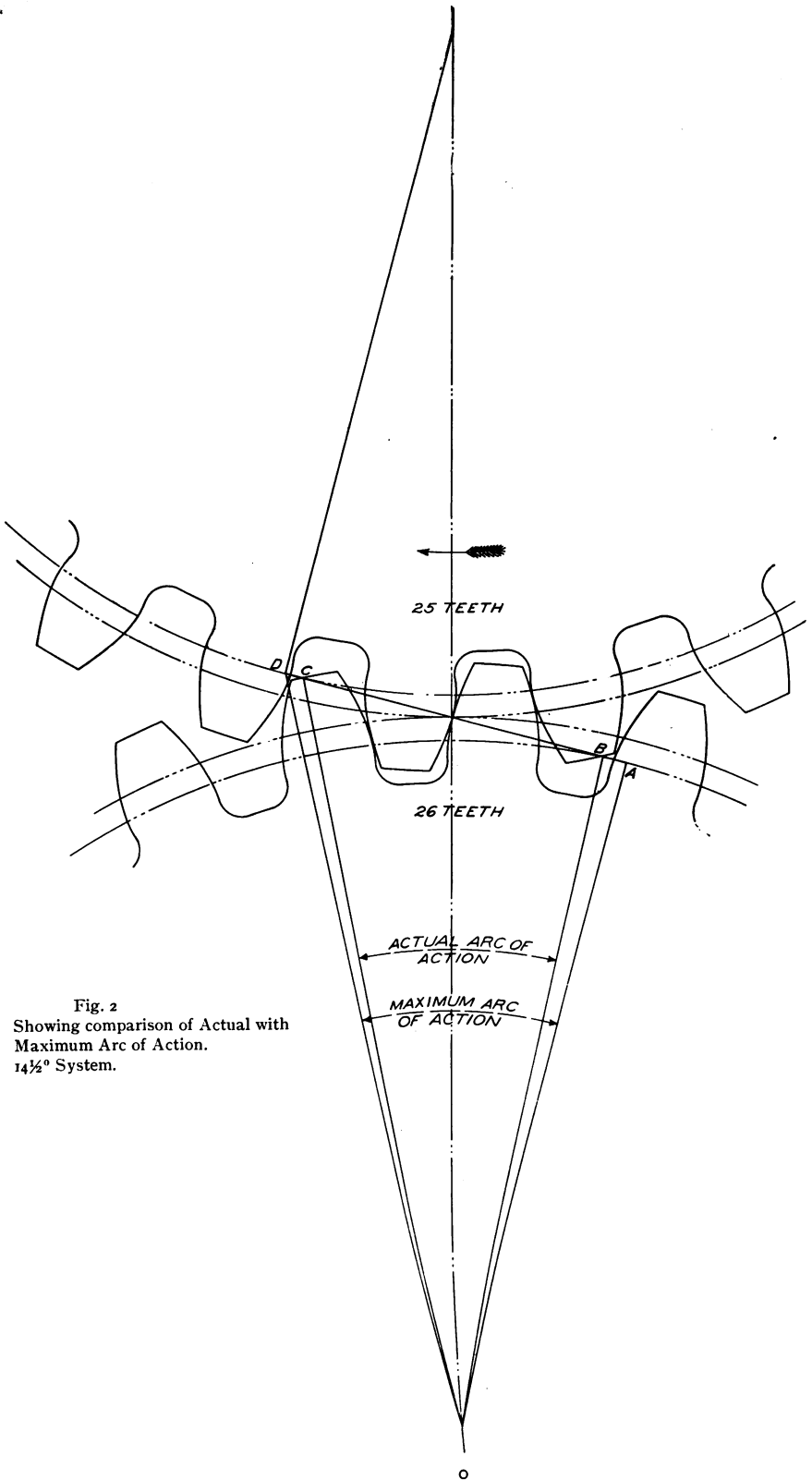


Fig. 2
 Showing comparison of Actual with
 Maximum Arc of Action.
 $14\frac{1}{2}^\circ$ System.

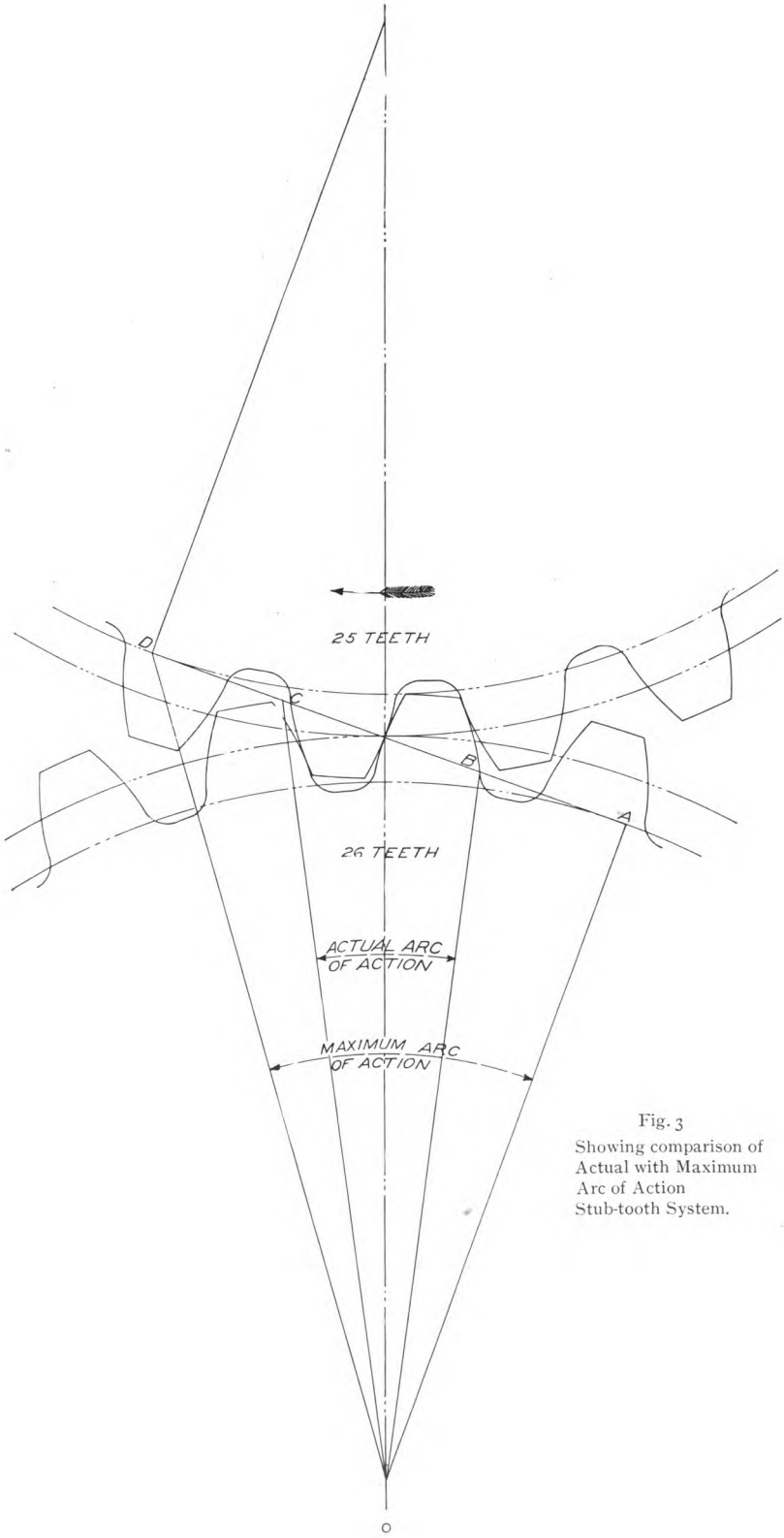


Fig. 3
 Showing comparison of
 Actual with Maximum
 Arc of Action
 Stub-tooth System.

In Figures 2 and 3 are shown comparisons of the tooth form of a gear of the standard and of the Stub-tooth, the driver having 25 and the driven 26 teeth. If, in the diagrams, the gears are supposed to rotate in the direction of the arrow, the theoretical action begins at A and ends at D, the line A D being termed the "line of action." It is obvious, however, that the actual action can only begin at B, where the outside diameter of the upper gear intersects the

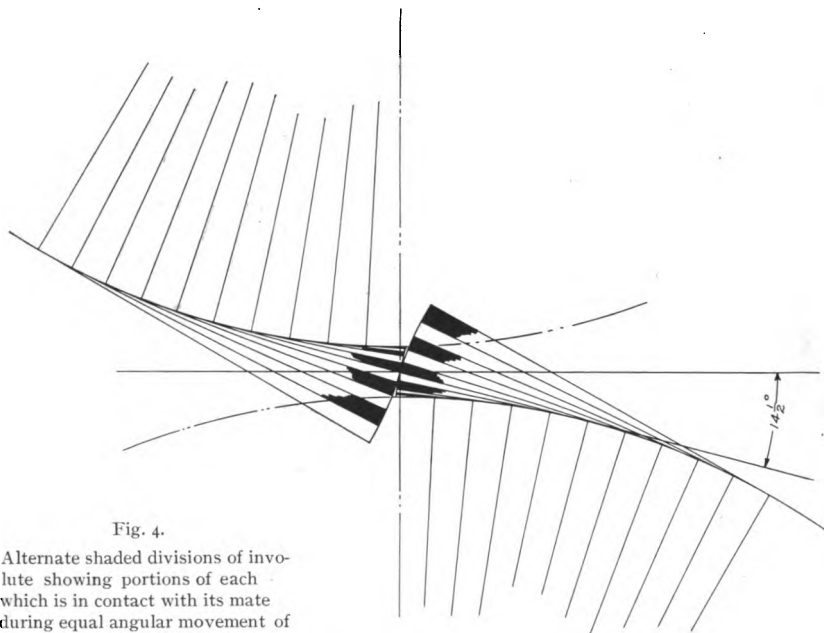


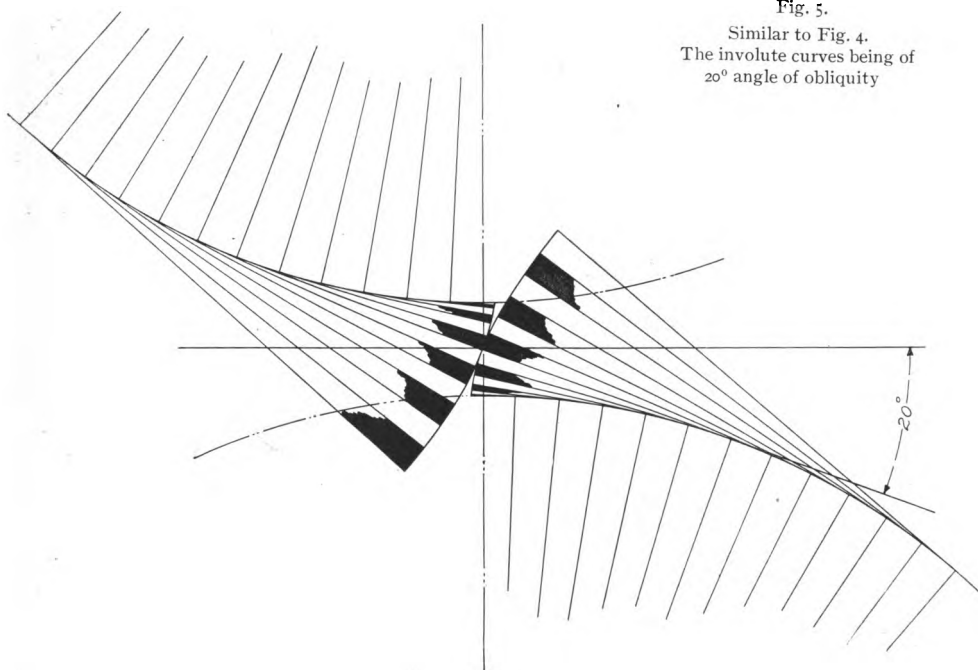
Fig. 4.

Alternate shaded divisions of involute showing portions of each which is in contact with its mate during equal angular movement of the gears. $14\frac{1}{2}^\circ$ angle of obliquity.

line A D, and ends at the corresponding point C. Drawing the radial lines from these points to the center O, the triangle A O D defines the maximum or the greatest possible arc of action, and B O C the actual arc of action.

In Figure 4 is shown an involute curve of $14\frac{1}{2}$ degrees obliquity of each of the gears, the curves being of sufficient length to cover the maximum arc of action, and drawn to the same scale as Figures 2 and 3. The alternately shaded divisions of the curves show the portion of each that is in contact with its mate during an equal angular movement of the gears.

Fig. 5.
Similar to Fig. 4.
The involute curves being of
 20° angle of obliquity



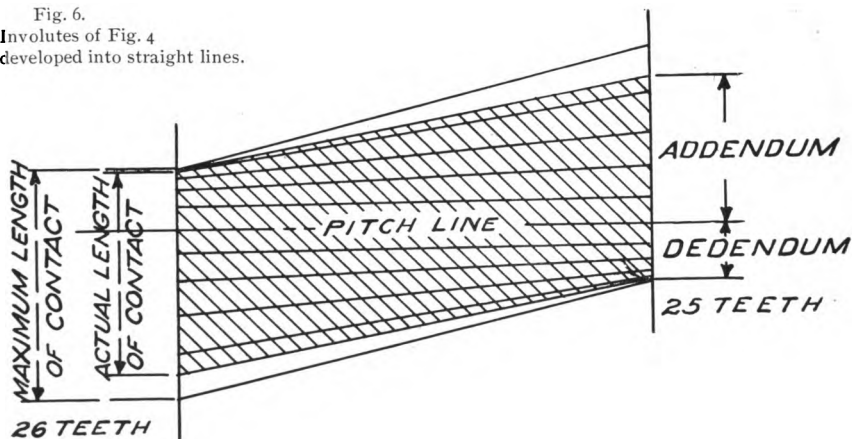
In Figure 5 is seen a similar diagram for a tooth having an angle of obliquity of 20 degrees.

In Figures 6 and 7 we have the same involute curves, developed into straight lines, the points corresponding to the divisions of Figures 4 and 5 being connected by cross lines.

To one who has labored under the impression that if the involute curves of a pair of gears are correct, the action is nearly a rolling one, a comparison of these diagrams will be both interesting and instructive.

It will be noted that although the divisions of the base circles are equal, those of the involute decrease as the base is approached, showing that the wear is concentrated at this point. When, however, it is considered that the contact is between the flank of the tooth and the point of its mate, it is seen that the condition is not ideal.

Fig. 6.
Involute of Fig. 4
developed into straight lines.



STANDARD TOOTH
 $14\frac{1}{2}^\circ$ ANGLE OF OBLIQUITY

By a comparison of Figures 6 and 7 note that the portion of actual contact, denoted by the shaded part, includes in the case of the $14\frac{1}{2}$ degree tooth cross lines that have a considerable angularity showing an excessive sliding action, while the corresponding lines of 20 degree teeth,

are nearly parallel, denoting that the action is nearly a rolling one.

Again referring to Figures 2 and 3 as showing a comparison of the two systems, we note two points in which they differ.

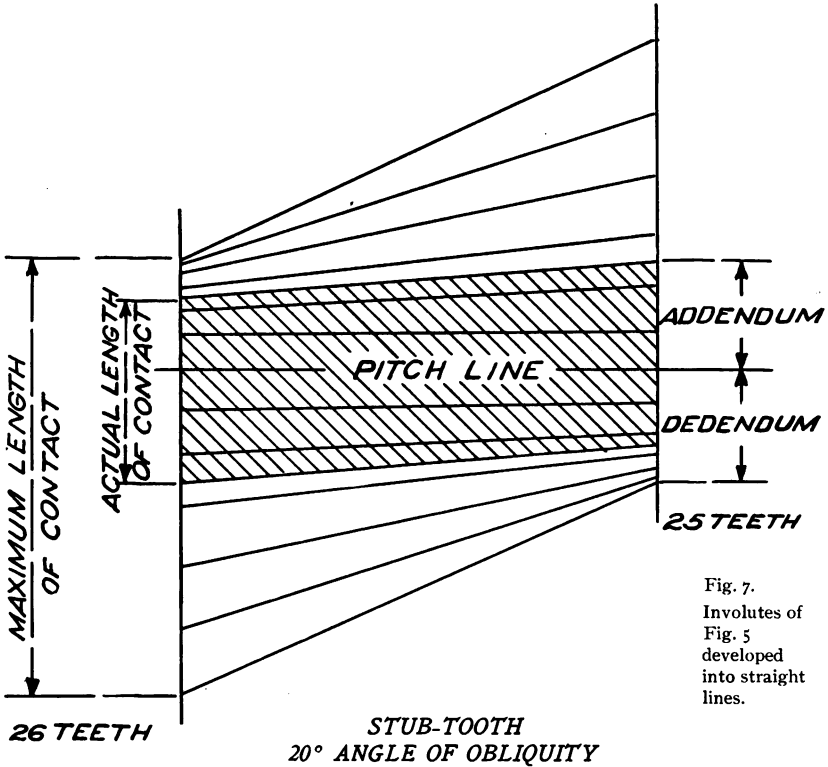


Fig. 7.
Involute of
Fig. 5
developed
into straight
lines.

First, on account of the greater angle of the line of action of the Stub-tooth, the maximum arc of action is much increased.

Second, the ratio of the *actual* to the *maximum* arc of action of the Stub-tooth is much less than in the standard.

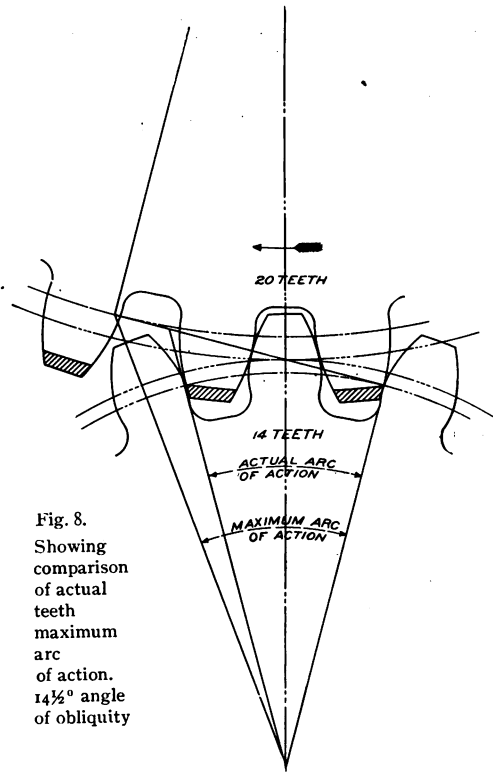


Fig. 8.
Showing
comparison
of actual
teeth
maximum
arc
of action.
 $14\frac{1}{2}^\circ$ angle
of obliquity

This latter point is a very important one, as we thus eliminate contact at both ends of the line of action. When we realize that this is the portion of the action in which the greater part of the sliding takes place, with its inevitable wear, we see that it is a good thing to cut it out if possible. The point of the tooth which wears out the flank of its mate is removed with the adoption of the Stub-tooth and this reduces the friction while increasing the efficiency. A comparison of Figures 6 and 7 shows that the action of the Stub-tooth is as nearly a rolling one as it is possible to obtain.

In Figures 8 and 9 are shown a pinion of 18 teeth engaged with a gear of 33 teeth and in Figures 10 and 11 the development of the involutes shows a comparison of the sliding action of the teeth made on the two systems. As the construction is the same as in the other case we have omitted the diagrams similar to Figures 6 and 7, from which Figures 10 and 11 were derived, and only the results are given.

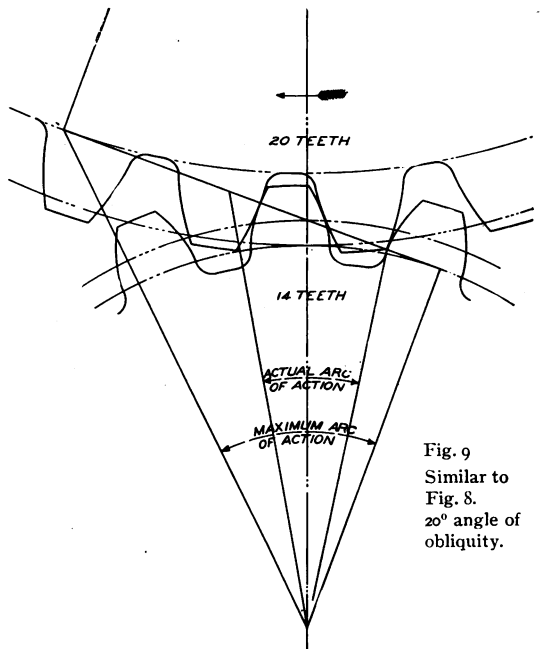


Fig. 9
Similar to
Fig. 8.
 20° angle of
obliquity.

Fig. 10.
 Involute of
 Fig. 8
 developed into
 straight lines.

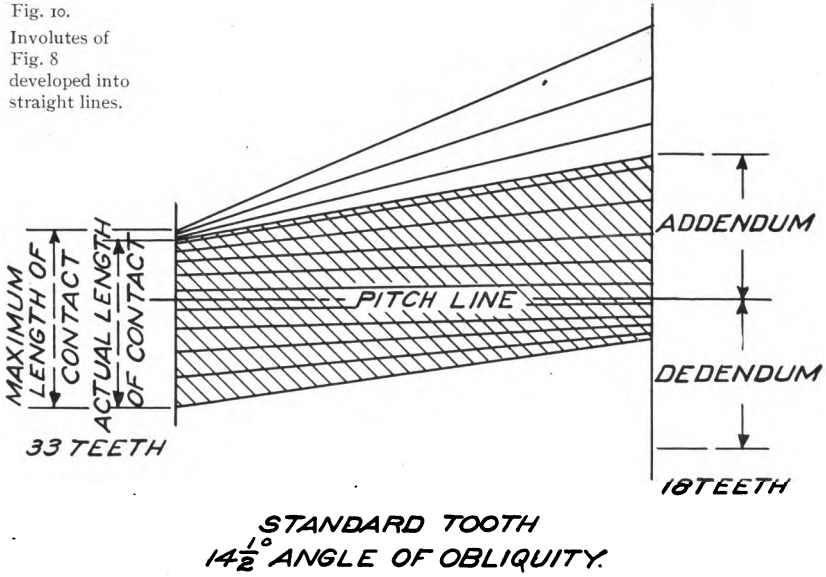
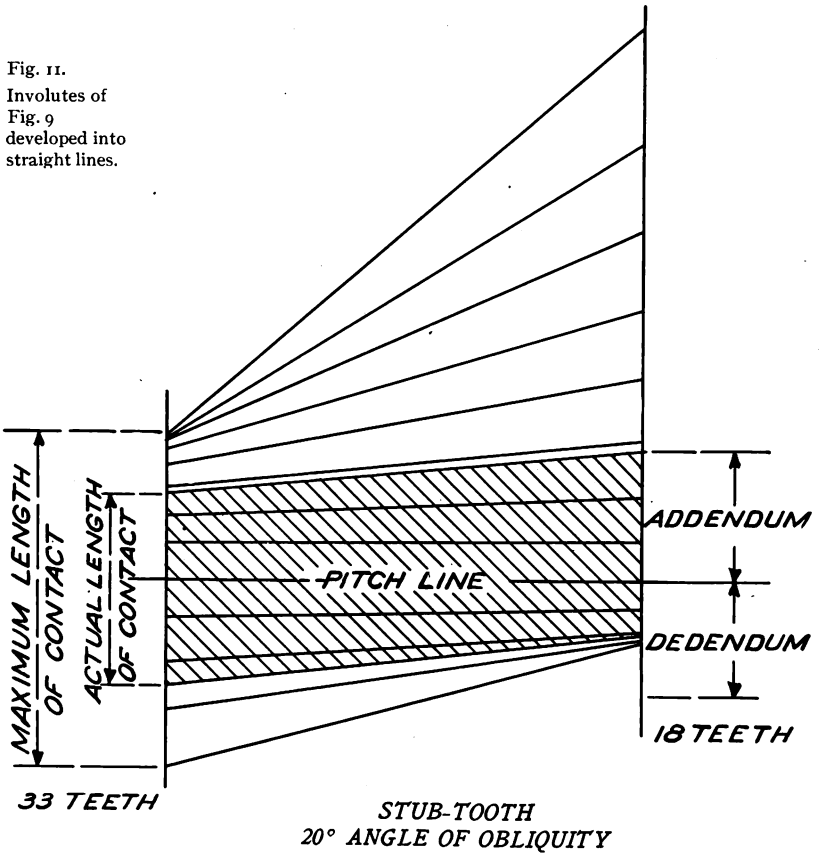


Fig. 11.
 Involute of
 Fig. 9
 developed into
 straight lines.



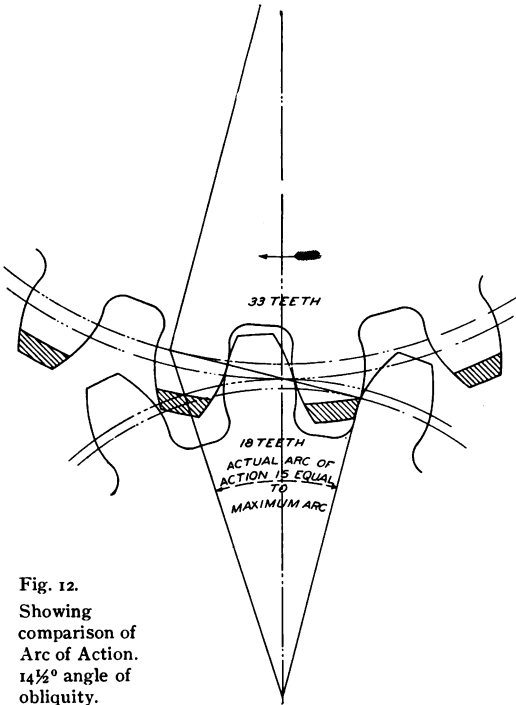


Fig. 12.
Showing
comparison of
Arc of Action.
 $14\frac{1}{2}^\circ$ angle of
obliquity.

account of the diameter of the base line of the pinion approaching so closely that of the pitch line, the entire length of the gear tooth does not have contact with the pinion. In order to accommodate this useless length of gear tooth the tooth of the pinion is materially and unnecessarily weakened. The shaded portion shows this length to be about 30 per cent of the addendum in one case and about 40 per cent in the other.

Figures 12 to 15 inclusive, show diagrams of the combination of 14 and 20 teeth, these being the driver and the intermediate gears of the gears for obtaining the reverse. In these cases, as in the one first considered, the sliding motion is largely eliminated from the Stub-tooth gear, due to the combination of the short tooth and the increased pressure angle. It will however be noted that in the case of the standard tooth, on

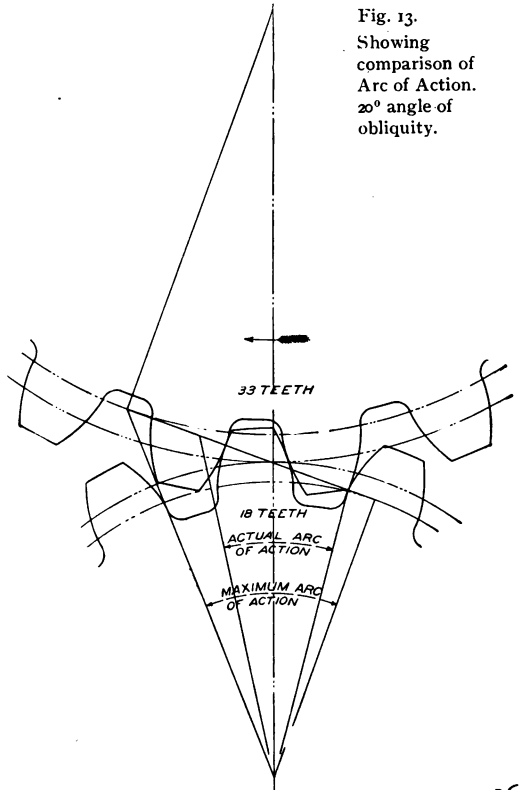
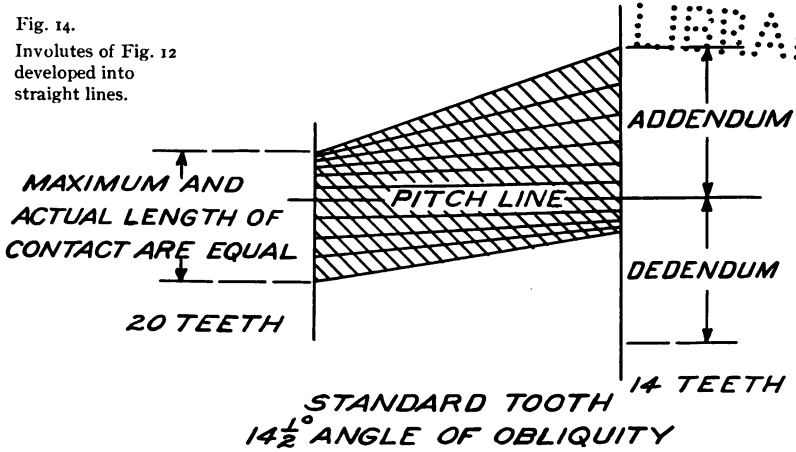


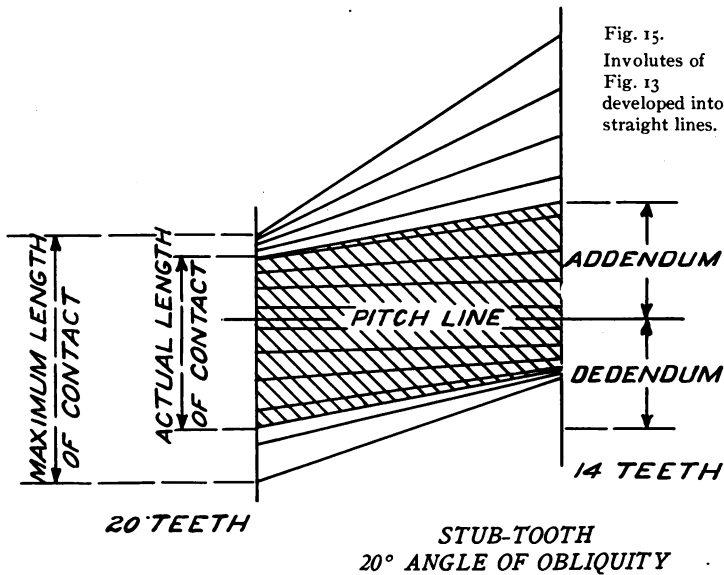
Fig. 13.
Showing
comparison of
Arc of Action.
 20° angle of
obliquity.

Fig. 14.
Involutes of Fig. 12
developed into
straight lines.



The action of the standard tooth at the base line, is that of a stone boat being dragged over the ground while the action of the Stub-tooth can be compared with the same stone boat mounted on wheels.

Fig. 15.
Involutes of
Fig. 13
developed into
straight lines.



THE
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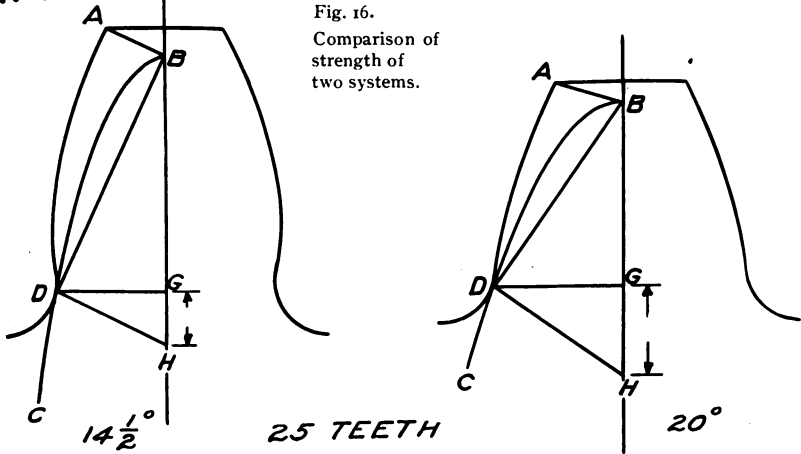


Fig. 16.
 Comparison of
 strength of
 two systems.

It is of course impossible to entirely eliminate the wear between the teeth of gears working under a load. But if the wear can be evenly distributed over the entire working face of the tooth, the correct form of tooth is retained indefinitely and a worn out gear should, aside from the excessive backlash, run as well as a new gear. And if this wear can be evenly distributed, the durability of any gear will be increased many times.

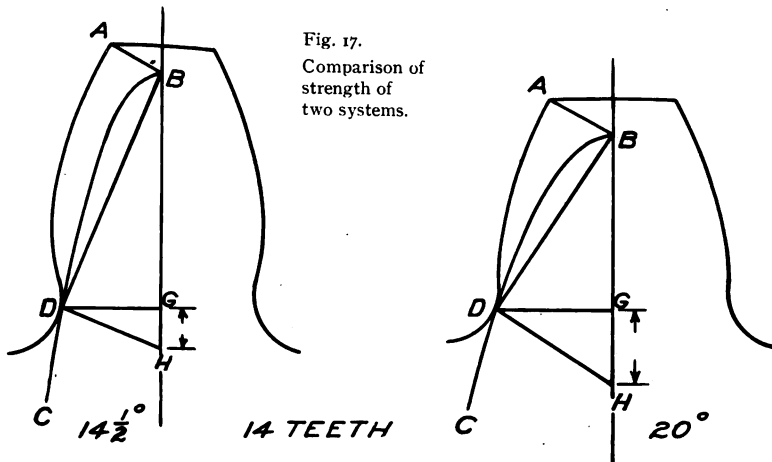
We have so far discussed only the points of efficiency and durability but there is another advantage of the Stub-tooth over the standard form, and one which some might think entitled to first consideration, especially in the transmission of any considerable amounts of power; this is the advantage of greatly increased strength.

Because of the fact that the tooth has been shortened and at the same time widened at its base, there is a very substantial gain in strength which in some cases amounts to nearly 100 per

cent and the fact that we can many times practically double the strength of a tooth is certainly worthy of serious consideration. In Figures 16 to 18 are shown enlarged sections of the different combinations of gears, showing the comparative strength of each form by the well known graphical method of Wilfred Lewis.

In laying out these diagrams we first draw the normal of the involute $A B$ from the extreme point of the tooth. From the point B , where the normal intersects the center line, erect the parabola $B C$, with its base tangent to the flank of the tooth and indicating D as the weakest section. Draw $D G$ at right angles to the center line and connect B and D . Then draw $D H$ at right angles to $B D$, intersecting the center line at H . In this construction $G H$ may be taken as the measure of the strength of the tooth.

A comparison of these diagrams of both the standard and Stub-teeth in this way, shows an increase of strength in these cases of 80, 77 and 71 per cent for these three combinations. It



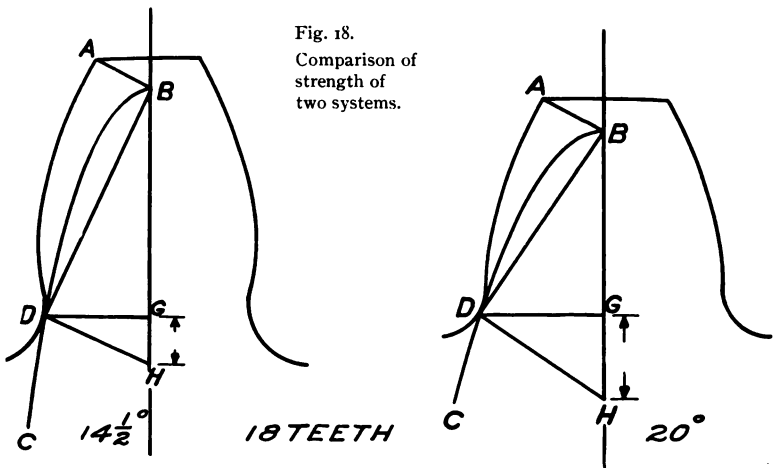


Fig. 18.
Comparison of
strength of
two systems.

should also be noted that the gain in strength is greatest in the case of the smallest pinion, which is a very great advantage, for as a chain is no stronger than its weakest link so a train of gears is no stronger than the teeth of its smallest pinion. This, in addition to the clearly demonstrated fact of easier running and reduced wear, should command careful consideration for the form of tooth here advocated.

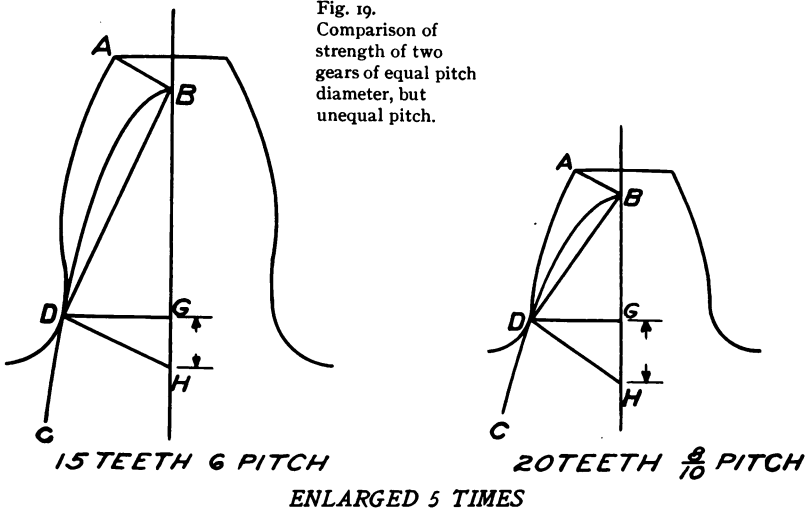
Figure 19 shows a comparison between a pinion having 15 teeth, 6 pitch and $14\frac{1}{2}$ degree pressure angle and one with 20 teeth, 8/10 pitch and Stub-tooth with its 20 degree pressure angle, both having the same pitch diameter. A comparison of the length of the line G H shows that the Stub-tooth, although much shorter, has 20 per cent greater strength and that while the bearing surface *per tooth* is shorter, the *total* area of bearing surface is 6 per cent greater.

This is not given as a suggestion that the pitch be reduced to secure an increase in strength of gears, as we have no desire to appear radical in

this matter, but it is shown merely to show one of the possibilities of the Stub-tooth.

Some of those who are not entirely familiar with this system of gear teeth have made the mistake of thinking that it consists simply of the shorter tooth than the standard form, while retaining the same pressure angle, and have therefore opposed it on the ground that the arc of action, in the case of a small pinion, is not equal to the pitch arc, and that the action is therefore not continuous because one tooth is out of action before the next tooth takes up the load.

It should be thoroughly understood, and we wish to emphasize the fact as much as possible, that the increased angle of obliquity is an essential and vital part of the Stub-tooth system and that with this increased angle, the arc of action is as long as that of the $14\frac{1}{2}$ degree tooth.



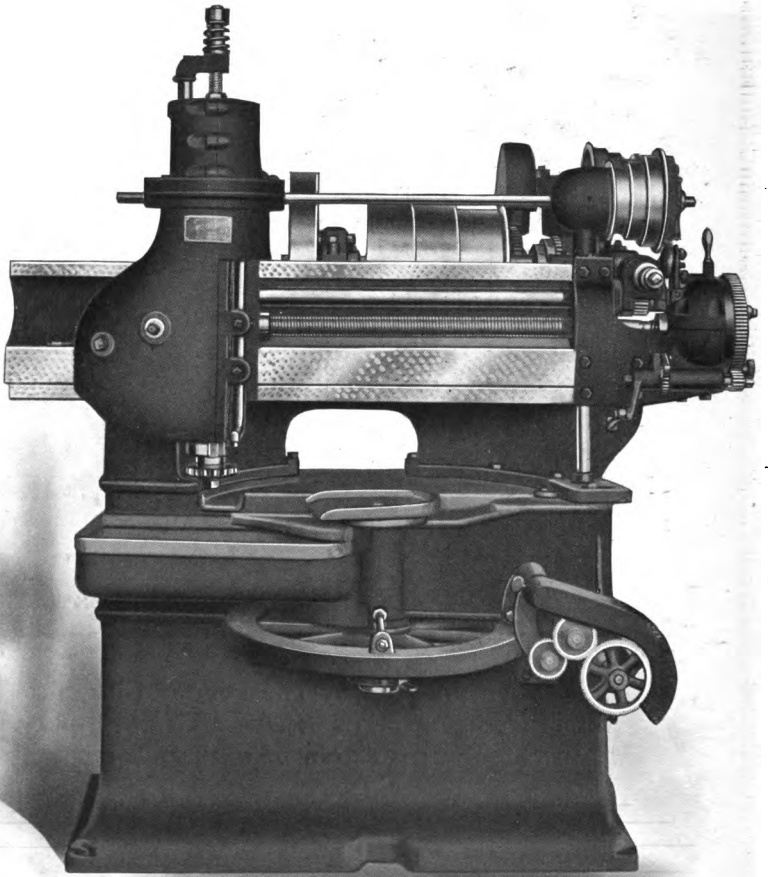
That this form of tooth is not a new and untried experiment is shown by the fact that it was first introduced by the Fellows Gear Shaper Company in 1899, and that its use has steadily increased until today it has been largely adopted both in this country and abroad until fully one-third of the cutters made for their Gear Shapers are now of the Stub-tooth form.

Is this not the best of evidence that it has real merit in actual practice and that it is well worth while investigating still further if you still have any doubts as to the wisdom of adopting it in your work?

TABLE OF TOOTH DIMENSIONS
OF THE
STUB-TOOTH GEAR.

PITCH	THICKNESS ON PITCH LINE	ADDENDUM	DEDENDUM
4/5	.3925	.200	.250
5/7	.314	.1429	.1785
6/8	.2617	.125	.1562
7/9	.2243	.111	.1389
8/10	.1962	.100	.125
9/11	.1744	.0909	.1137
10/12	.157	.0833	.1042
12/14	.1308	.0714	.0893

These cutters are made by The Fellows Gear Shaper Co. for use only upon the Fellows Gear Shaper, an illustration of which is shown upon the following page.



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THE 36 INCH FELLOWS GEAR SHAPER